Received: *March 15, 1999*, Accepted: *June 30, 1999*

$N=(0,4)$ quiver SCFT$_2$ and supergravity on $AdS_3 \times S^2$

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Abstract: We study the proposed duality between the 5-dimensional supergravity/superstring on $AdS_3 \times S^2$ and the 2-dimensional $N=(0,4)$ SCFT defined on the boundary of AdS-space. We construct explicitly the $N=(0,4)$ SCFT by imposing the "quiver projection" developed by Douglas-Moore [14] on the $N=(4,4)$ SCFT of symmetric orbifold, which is proposed to be the dual of the 6-dimensional supergravity/superstring on $AdS_3 \times S^3$. We explore in detail the spectrum of chiral primaries in this "quiver SCFT$_2$". We compare it with the Kaluza-Klein spectrum on $AdS_3 \times S^2$ and check the consistency between them. We further emphasize that orbifolding of bulk theory should *not* correspond to orbifolding of the boundary CFT in the usual sense of two dimensional CFT, rather corresponds to the quiver projection. We observe that these are not actually equivalent with each other when we focus on the multi-particle states.

Keywords: Conformal Field Models in String Theory, String Duality, D-branes, M-Theory.

JHEP06(1999)035

Contents

1. Introduction

AdS_3/CFT_2-duality has been understood most successfully among the several proposals of AdS_{d+1}/CFT_d-duality [1, 2, 3] in general dimensions. This is due to the fact that the two dimensional conformal symmetry is very powerful to analyse the spectrum, and three dimensional gravity with a negative cosmological term can be quantized by recasting it to Chern-Simons theory [4, 5]. Among other things, one of the most remarkable success is the complete agreement between the BPS Kaluza-Klein (KK) spectrum of supergravity theory (SUGRA) on $AdS_3 \times S^3 \times M^4$ (with $M^4 = T^4$ or $K3$) and the spectrum of chiral primary states of $N = (4, 4)$ SCFT_2 with the target manifold $\mathrm{Sym}^N(M^4)$ [6, 7, 8]. However, analyses on the models with less supersymmetries are still far from complete. In this paper we shall focus on the correspondence between the 5-dimensional SUGRA on $AdS_3 \times S^2$ and the $N = (0, 4)$ SCFT_2 defined on the boundary of AdS_3.

The most general examples which realize this geometry in the near horizon limit are given by considering the M-theory compactified on some Calabi-Yau 3-fold M^6 (including the cases $M^6 = T^6$, $T^2 \times K3$) [1, 9, 10], and we can construct the boundary

$N = (0,4)$ SCFT_2 by making M5-brane wind around a suitable supersymmetric 4-cycle P in M^6 with non-vanishing triple self-intersection: $P^3 \neq 0$. Although it is known that these CFT_2 can thoroughly reproduce the correct Bekenstein-Hawking entropy in the microscopic level, it is still a non-trivial problem to analyse the detailed spectra of chiral primaries. The present work is motivated by this fact.

To resolve the difficulty to study the $N = (0,4)$ SCFT_2 derived from this configuration of M5, we shall choose another way. That is, we will work out in a T-dualized framework. We focus on the limited examples $M^6 = T^6$ or $T^2 \times K3$ through this paper, and can easily find that the above brane configuration of M5 is equivalent by T-duality to the system of D1-D5-TN5 in IIB string theory. (Here "TN5" means "Taub-NUT 5-brane", also sometimes called "KK 5-brane", which is defined as a Taub-NUT solution in the transversal 4-dimensional space). We take this brane configuration as the starting point to analyse the $N = (0,4)$ SCFT_2 following the several authors [11, 12, 13]. The advantage to do so is as follows: the D1-D5 system leads to the well-known $N = (4,4)$ SCFT_2 we already mentioned, which is finely understood at least as a CFT of symmetric orbifold, and putting these branes at the location of singularity of Taub-NUT ($\sim$ ALE) can be also described by the familiar procedure [14], which is realized by imposing suitable projection on the Chan-Paton (CP) indices. We shall call this projection as the "quiver projection" through this paper. In this manner we can reduce our problem to a "tractable" one. We will *define* the Hilbert space of the desired $(0,4)$ SCFT_2 by directly imposing the quiver projection on that of the $(4,4)$ SCFT_2 as a symmetric orbifold theory. Although the "quiver $(0,4)$ SCFT_2" defined in this way describes only the symmetric orbifold point of the moduli space, it is expected that we can analyse the spectrum which are *stable under marginal deformations* by means of this SCFT_2. After carrying out the detailed analyses on chiral primaries in the quiver SCFT_2, we will find out the complete agreement between the chiral primary states and the spectrum of KK excitations in the $5d$ SUGRA on $AdS_3 \times S^2$ under the Maldacena limit.

This paper will be organized as follows.

In section 2 we give a short review about some known results of the 5-dimensional SUGRA with $AdS_3 \times S^2$-geometry. Particularly, we demonstrate the brane configuration on which our construction of the boundary SCFT_2 is based.

Our main discussions will be presented in sections 3 and 4. We there give our proposal of the boundary $N = (0,4)$ SCFT_2 as the one constructed by making use of the quiver techniques [14], and explore the detailed spectrum of the chiral primary states. Our analysis of spectrum consists of three parts; the single particles in the untwisted sector, the single particles in the various twisted sectors, and lastly we discuss the spectrum of the general multi-particle states. We will find out the complete agreement between the BPS KK spectrum of the $AdS_3 \times S^2$-SUGRA and the chiral primaries in the quiver $(0,4)$ SCFT_2. We will also emphasize, especially in relation to the analysis on multi-particles, that our SCFT_2 defined by the quiver techniques is *not* equivalent with that defined by the usual $\mathbb{Z}_q$-orbifoldization. We observe that the results of SUGRA support the quiver $(0,4)$ SCFT_2, not the one obtained by $\mathbb{Z}_q$-orbifoldization.

In section 5 we summarize our results and give a few comments about open problems.

2. $5d$ SUGRA on $AdS_3 \times S^2$

In this section we briefly survey some known results with respect to the 5-dimensional SUGRA on $AdS_3 \times S^2$ for later convenience. We begin with the simplest example: M-theory compactified on $M^6 = T^6$. We can also present the similar arguments for the case of $M^6 = T^2 \times K3$.

For the 6-torus $M^6 = T^6$, the M5-brane wrapped around a suitable SUSY 4-cycle which we previously mentioned means nothing but the three intersecting 5-branes [15]. Let $(x^5, x^6, \ldots, x^{11})$ be the coordinates along the internal space M^6, and consider the following 5-brane configuration (we of course regard x^0 as the time coordinate)

- q_1 M5s extending along the (016789),
- q_2 M5s extending along the (0156711),
- q_3 M5s extending along the (0158911).

The 11-dimensional metric corresponding to this brane configuration is given as follows [15]

$$\begin{aligned} ds_{11}^2 = (H_1H_2H_3)^{2/3}\Big\{&(H_1H_2H_3)^{-1}\Big(-(dx^0)^2 + (dx^1)^2\Big) + \\ &+ (H_1H_2)^{-1}\Big((dx^6)^2 + (dx^7)^2\Big) + (H_2H_3)^{-1}\Big((dx^5)^2 + (dx^{11})^2\Big) + \\ &+ (H_3H_1)^{-1}\Big((dx^8)^2 + (dx^9)^2\Big) + \Big(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\Big)\Big\}, \end{aligned} \tag{2.1}$$

where (r, θ, ϕ) stands for the spherical coordinates along the (x^2, x^3, x^4)-directions which are transversal to all of the branes, and the harmonic functions H_i in this transversal space should be defined as

$$H_i(r) \stackrel{\text{def}}{=} 1 + \frac{2\pi l_{\mathrm{P}} q_i}{\omega_2 r v^{1/3}} \equiv 1 + \frac{l_P q_i}{2 r v^{1/3}}\,. \tag{2.2}$$

In the expressions of these harmonic functions l_{P} means the 11-dimensional Planck length (which is defined, in our convention, by $G_{11} = 16\pi^7 l_{\mathrm{P}}^9$ from the 11-dimensional Newton constant G_{11}), $\omega_2 \equiv 4\pi$ means the area of unit 2-dimensional sphere and we set the volume of the internal space M^6 as $\mathrm{Vol}(M^6) = v(2\pi l_{\mathrm{P}})^6$. Under the Maldacena limit, that is,

$$D \equiv q_1 q_2 q_3 \longrightarrow \infty\,, \quad \text{with } l \equiv l_{\mathrm{P}} \left(\frac{D}{v}\right)^{1/3} \gg l_{\mathrm{P}} \text{ fixed}\,, \tag{2.3}$$

this metric reduces to that of $AdS_3 \times S^2 \times M^6$

$$ds^2 = \left(\frac{l}{u}\right)^2 du^2 + \left(\frac{u}{l}\right)^2 dx^+ dx^- + \frac{l^2}{4}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + ds^2(M^6)\,, \tag{2.4}$$

where we set $u = \sqrt{2lr}$, $x^{\pm} = \pm x^0 + x^1$.

It is proposed [1] that the boundary CFT for this example should correspond to an $N = (0,4)$ SCFT_2 defined on the triple intersection of M5s (x^0, x^1-directions). This CFT is the simplest example among those studied in [9, 10] and was first considered in [15] in the context of the microscopic analysis on black-hole entropy. Its central charge can be evaluated under the "large N limit" (2.3) by counting the bosonic and fermionic degrees of freedom. We can also evaluate it by means of the famous formula by Brown-Hennaux [16] $c = 3l/2G_3$ according to the discussion given in [17]. Recalling that the 3-dimensional Newton constant G_3 is computed as

$$\begin{aligned}\frac{1}{16\pi G_3} &= \frac{1}{16\pi G_{11}} \times \mathrm{Vol}(M^6) \times \mathrm{Vol}(S^2) = \\ &= \frac{1}{(2\pi)^8 l_{\mathrm{P}}^9} \times (2\pi l_{\mathrm{P}})^6 v \times 4\pi \left(\frac{l}{2}\right)^2 = \frac{v^{1/3} D^{2/3}}{4\pi l_{\mathrm{P}}}\,,\end{aligned} \tag{2.5}$$

we can immediately obtain the familiar central charge $c = 6D$ which reproduces the correct Bekenstein-Hawking entropy.

Although this evaluation seems satisfactory, this is not the whole story in the AdS_3/CFT_2-correspondence. This SCFT defined on the triple intersection of 5-branes is not so powerful to analyse the detailed structure of spectrum. Therefore, we shall take a different framework. It is easy to find the following equivalence by T-duality

$$\begin{aligned} M/\big(T^6(5678911)\big) &\begin{cases} q_1\mathrm{M5}(016789) \\ q_2\mathrm{M5}(0156711) \\ q_3\mathrm{M5}(0158911) \end{cases} \cong IIA/\big(T^5(56789)\big) \begin{cases} q_1\mathrm{NS5}(16789) \\ q_2\mathrm{D4}(01567) \\ q_3\mathrm{D4}(01589) \end{cases} \\ &\xrightarrow{T^5T^6T^7} IIB/\big(\tilde{T}^5(56789)\big) \begin{cases} q_1\mathrm{TN5}(016789) \\ q_2\mathrm{D1}(01) \\ q_3\mathrm{D5}(016789). \end{cases} \end{aligned} \tag{2.6}$$

In the last line "TN5(016789)" strictly means that the transversal space (x^2, x^3, x^4, x^5) is the Taub-NUT space with the monopole charge q_1 (the S^1-fiber is along x^5). Lastly, let us consider the decompactification limit of Taub-NUT space, which means (x^2, x^3, x^4, x^5) becomes the ALE space $\mathrm{ALE}(A_{q_1-1}) \sim \mathbb{C}^2/\mathbb{Z}_{q_1}$.

In this way we have led to the system of $Q_1 \equiv q_1 q_2$ D1 and $Q_1 \equiv q_1 q_3$ D5 wrapped around $T^4(6789)$, which are located at the singularity of the orbifold $\mathbb{C}^2/\mathbb{Z}_{q_1}$ (the 2345th-directions). (The origin of the extra factor "q_1" in the brane charges is the existence of mirror images by the orbifold $\mathbb{Z}_{q_1}$-action.) This brane configuration leads to the geometry $AdS_3 \times S^3/\mathbb{Z}_{q_1} \times T^4$. Some analyses on this AdS_3 geometry were given in [11] and, the studies from the viewpoints of string theory along the same line as [18] were presented in [12]. However, we shall take a route which is different from them and is rather analogous to the works [19, 20, 21, 22]. The familiar techniques to put D-branes at the locations of orbifold singularities were invented by Douglas-Moore [14] and we shall call this procedure the "quiver projection" in this paper. In the next section we will construct the boundary SCFT by means of this projection from the well-known $(4,4)$-SCFT corresponding to the Q_1D1-Q_5D5 system which has the near-horizon geometry $AdS_3 \times S^3 \times M^4$. Furthermore, we will explore the spectrum of chiral primaries in this "quiver $(0,4)$-SCFT", and will observe how we can get the complete agreement with the KK-spectrum of $5d$ SUGRA on $AdS_3 \times S^2$. Here we should comment on the work [13]: it is there carefully discussed that the perturbative degrees of freedom in such a quiver SCFT as above are completely reproduced also in the U-dualized framework of intersecting branes by incorporating the non-perturbative BPS states of the string junctions [23], although the concrete analysis on the chiral primaries as in the present paper are not given.

In the case of $M^6 = T^2 \times K3$ we can likewise T-dualize the orginal system, at least for the elliptic K3.[1] We can again arrive at the Q_1D1-Q_5D5 system such that D5s are wrapped around K3 and all of the branes are located at the orbifold point of $\mathbb{C}^2/\mathbb{Z}_{q_1}$. We can also make up the boundary $(0,4)$ SCFT_2 by the quiver projection, but there is one subtle point. It is known [9, 10] that the central charge of this boundary CFT suffers quantum correction, when M^6 is a curved manifold. It seems difficult to realize this correction in the framework of our quiver SCFT_2. However, since this correction becomes negligible if taking the Maldacena limit, we can expect it works in the K3 case, too.

To close this section let us present the BPS KK-spectra on $AdS_3 \times S^2 \times M^6$ SUGRA ($M^6 = T^6$, $T^2 \times K3$) which are given in [24, 6, 12, 25]. These spectra should be encoded to the family of the short representations of super Lie group $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SU}(1,1|2)_R$ describing the symmetry of $AdS_3 \times S^2$-SUGRA. The general irreducible representations of $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SU}(1,1|2)_R$ are classified by three half integers $(h, \bar{h}, \bar{j}) \in (1/2)\mathbb{Z}_{\geq 0} \times (1/2)\mathbb{Z}_{\geq 0} \times (1/2)\mathbb{Z}_{\geq 0}$. h denotes the highest weight of $\mathrm{SL}(2,\mathbb{R})_L$-factor and $\bar{h}, \bar{j}$ correspond to those of $\mathrm{SU}(1,1|2)_R$. The BPS condition can

[1] Of course, the moduli space of elliptic K3 is merely a subspace of the full moduli space of K3. But our concern in this paper is to evaluate the spectrum that does not depend on the generic value of moduli.

simply read as $\bar{h} = \bar{j}$. We here express the each short representation by the notation $(\bar{j}; s)$, where $s \stackrel{\text{def}}{=} h - \bar{h} \equiv h - \bar{j}$ denotes the "spin", in the similar manner as [6]. Actually s is equal to the spin of the lowest component of the BPS multiplet in the AdS_3-space.

Under these preparations the BPS KK-spectrum for $AdS_3 \times S^2 \times T^6$ can be written as follows

$$
\begin{aligned}
\mathcal{H}_{\text{SUGRA}}(T^6) &= \mathcal{H}^{(+)}_{\text{SUGRA}}(T^6) \oplus \mathcal{H}^{(-)}_{\text{SUGRA}}(T^6)\,, \\
\mathcal{H}^{(+)}_{\text{SUGRA}}(T^6) &= (1;2) + 15(1;1) + 14(1;0) + \\
&\quad + \sum_{n=2}^{\infty} \Big\{ (n;2) + 15(n;1) + 15(n;0) + (n;-1) \Big\}, \\
\mathcal{H}^{(-)}_{\text{SUGRA}}(T^6) &= 6\left(\frac{1}{2};\frac{3}{2}\right) + 14\left(\frac{1}{2};\frac{1}{2}\right) + \\
&\quad + \sum_{n=2}^{\infty} \left\{ 6\left(n-\frac{1}{2};\frac{3}{2}\right) + 20\left(n-\frac{1}{2};\frac{1}{2}\right) + 6\left(n-\frac{1}{2};-\frac{1}{2}\right) \right\}.
\end{aligned}
\tag{2.7}
$$

In these expressions $\mathcal{H}^{(+)}_{\text{SUGRA}}$ and $\mathcal{H}^{(-)}_{\text{SUGRA}}$ correspond to the spaces of the multiplets in which lowest component is a boson and a fermion respectively.

For $AdS_3 \times S^2 \times T^2 \times K3$, we can also write down the spectrum in the similar manner

$$
\begin{aligned}
\mathcal{H}_{\text{SUGRA}}(T^2 \times K3) &= \mathcal{H}^{(+)}_{\text{SUGRA}}(T^2 \times K3) \oplus \mathcal{H}^{(-)}_{\text{SUGRA}}(T^2 \times K3), \\
\mathcal{H}^{(+)}_{\text{SUGRA}}(T^2 \times K3) &= (1;2) + 23(1;1) + 22(1;0) + \\
&\quad + \sum_{n=2}^{\infty} \Big\{ (n;2) + 23(n;1) + 23(n;0) + (n;-1) \Big\}, \\
\mathcal{H}^{(-)}_{\text{SUGRA}}(T^2 \times K3) &= 2\left(\frac{1}{2};\frac{3}{2}\right) + 42\left(\frac{1}{2};\frac{1}{2}\right) + \\
&\quad + \sum_{n=2}^{\infty} \left\{ 2\left(n-\frac{1}{2};\frac{3}{2}\right) + 44\left(n-\frac{1}{2};\frac{1}{2}\right) + 2\left(n-\frac{1}{2};-\frac{1}{2}\right) \right\}.
\end{aligned}
\tag{2.8}
$$

The mass spectrum of KK excitations is given from the values of "conformal weights" by the famous formula [2, 9, 6]

$$
l^2 m^2 = (h + \bar{h})(h + \bar{h} - 2) \equiv (2\bar{j} + s)(2\bar{j} + s - 2). \tag{2.9}
$$

3. Quiver $N = (0,4)$ SCFT$_2$ as boundary CFT

As was demonstrated in the previous seciton, the situation we should study is summarized as follows

- Q1 D1 extending along (x^0, x^1).

- Q5 D5 extending along $(x^0, x^1, x^6, x^7, x^8, x^9)$, and the space along (x^6, x^7, x^8, x^9) is $M^4 = T^4, K3$.

- the space (x^2, x^3, x^4, x^5) is the orbifold $\mathbb{C}^2/\mathbb{Z}_{q_1}$, and all of the branes sit at the orbifold singular point $x^2 = \cdots = x^5 = 0$.

Here we set $Q_1 \equiv q_1 q_2$, $Q_5 \equiv q_1 q_3$.

Notice that our orbifold group $\mathbb{Z}_{q_1}$ acts along the directions transverse to all of the branes. It is different from the situations intensively studied in [14], in which the orbifold group acts along (x^6, x^7, x^8, x^9)-directions and one can gain a natural interpretation of Kronheimer-Nakajima's theorem with respect to the instantons on ALE spaces [26] by means of the brane theory.

Nevertheless, we can apply the same techniques and can easily write down the "inner" and "outer" quiver diagrams to encode the matter contents. By this prescription we can obtain an $N = (0, 4)$ supersymmetric theory which is an example of the general models with $(0, 4)$ SUSY discussed in [27, 28], called "ADHM gauged linear σ-model". In principle the IR limit of its Higgs branch will provide us the desired $N = (0, 4)$ SCFT_2.

However, there are subtle points. In the quantum level it is not easy to determine exactly the low energy effective theory of the ADHM linear σ-model, because we only have less supersymmetries. Moreover, *even* in the cases with $N = (4, 4)$ SUSY, the famous proposal of the target space $\mathrm{Sym}^{Q_1 Q_5}(M^4)$ is still a conjecture. No one succeeded in justifying this proposal completely from the viewpoint of two-dimensional quantum gauge theory.[2]

This $(4, 4)$ σ-model on $\mathrm{Sym}^{Q_1 Q_5}(M^4)$ nonetheless has a great success in AdS_3/CFT_2-duality [30, 6, 7, 31, 8]. So it seems to be the model worth taking as a good starting point. In order to define our boundary $N = (0, 4)$ SCFT_2, we shall directly carry out the quiver projection on the $N = (4, 4)$ orbifold SCFT_2 on $\mathrm{Sym}^{Q_1 Q_5}(M^4)$ *in place of taking the IR limit of the ADHM linear σ-model.* This is our standpoint on which the analyses in this paper will be based, and let us first clarify the definition of Hilbert space of this theory.

3.1 Hilbert space of quiver SCFT_2

In this subsection we present the explicit definition of the quiver projection to the Hilbert space of $\mathrm{Sym}^{Q_1 Q_5}(M^4)$ SCFT_2.

For the time being we discuss the simpler example: $M^4 = T^4$. Let $\alpha, \dot{\alpha} = \pm$ be the spinor indices in the (x^2, x^3, x^4, x^5)-space and $a, \dot{a} = \pm$ be those in the (x^6, x^7, x^8, x^9)-space (compactified on T^4).

[2] In fact, there is a different proposal of the IR effective theory [29]. This has the same degrees of freedom, but a different discrete symmetry which should be gauged as orbifold CFT_2. So, it may lead to the different spectrum of chiral primaries.

The description of the $N=(4,4)$ SCFT$_2$ on $\mathrm{Sym}^{Q_1Q_5}(T^4)$ is elementary. We have $4Q_1Q_5$ numbers of free bosons $X^{a\dot{a}}_{(AI)}$ and free fermions $\psi^{\alpha\dot{a}}_{(AI)}$, $\bar{\psi}^{\dot{\alpha}\dot{a}}_{(AI)}$ ($A=0,\ldots,Q_1-1$ "color indices", $I=0,\ldots,Q_5-1$ "flavor indices") as the fundamental fields, which roughly correspond to the degrees of freedom of the "1–5" open strings. The superconformal symmetry is realized by the following currents (We only write the left-mover.)

$$
\begin{aligned}
T(z) &= -\frac{1}{2}\sum_{AI}\epsilon_{ab}\epsilon_{\dot{a}\dot{b}}\partial X^{a\dot{a}}_{(AI)}\partial X^{b\dot{b}}_{(AI)} - \frac{1}{2}\sum_{AI}\epsilon_{\alpha\beta}\epsilon_{\dot{a}\dot{b}}\psi^{\alpha\dot{a}}_{(AI)}\partial\psi^{\beta\dot{b}}_{(AI)}\,, \\
G^{\alpha a}(z) &= i\sum_{AI}\epsilon_{\dot{a}\dot{b}}\psi^{\alpha\dot{a}}_{(AI)}\partial X^{a\dot{b}}_{(AI)}\,, \\
J^{\alpha\beta}(z) &= \frac{1}{2}\sum_{AI}\epsilon_{\dot{a}\dot{b}}\psi^{\alpha\dot{a}}_{(AI)}\psi^{\beta\dot{b}}_{(AI)}\,.
\end{aligned}
\tag{3.1}
$$

These generate the $N=4$ (small) superconformal algebra (SCA) with central charge $c=6Q_1Q_5$. (The usual convention of SU(2)-current is $J^I(z)\equiv\sigma^I_{\alpha\beta}J^{\alpha\beta}(z)$ ($I=1,2,3$).) The subalgebra of "zero-modes" $\{L_{\pm1},L_0,G^{\alpha a}_{\pm1/2},J^{\alpha\beta}_0\}$ and their counterpart of right-mover composes the super Lie algebra $\mathrm{SU}(1,1|2)_L\times\mathrm{SU}(1,1|2)_R$ corresponding to the symmetry supergroup in the $AdS_3\times S^3$ geometry.

According to the general theory of orbifold CFT$_2$ [32], we have various twisted sectors labeled by the discrete symmetry group $S_{Q_1Q_5}$. Each of them is defined as the Fock space of bosonic and fermionic free fields with the twisted boundary conditions associated to $g\in S_{Q_1Q_5}$

$$
\Phi_{(AI)}(\sigma+2\pi)=\Phi_{g((AI))}(\sigma)\,,
\tag{3.2}
$$

where $\Phi_{(AI)}(\sigma)$ denotes $X^{a\dot{a}}_{(AI)}$, $\psi^{\alpha\dot{a}}_{(AI)}$, $\bar{\psi}^{\dot{\alpha}\dot{a}}_{(AI)}$. The total Hilbert space of this orbifold theory should have the next structure

$$
\mathcal{H}^{(4,4)}=\left(\bigoplus_{g\in S_{Q_1Q_5}}\mathcal{H}_g\right)^{S_{Q_1Q_5}}\,.
\tag{3.3}
$$

Here $\mathcal{H}_g$ ($g\in S_{Q_1Q_5}$) denotes the Fock space of the g-twisted fields, and the superscript "$S_{Q_1Q_5}$" indicates the $S_{Q_1Q_5}$-invariant subspace. This Hilbert space (3.3) can be also expressed [33] as follows

$$
\mathcal{H}^{(4,4)}=\bigoplus_{[g]\in Y_{Q_1Q_5}}\mathcal{H}_{[g]}\,.
\tag{3.4}
$$

Here the direct sum is taken over the set of all the conjugate class of $S_{Q_1Q_5}$ (which is identified with the set of Young tableaus of Q_1Q_5 boxes), and we set

$$
\mathcal{H}_{[g]}=\bigoplus_{h\in[g]}(\mathcal{H}_h)^{C(h)}\,,
\tag{3.5}
$$

where $C(h)\equiv\{g\in S_{Q_1Q_5}: gh=hg\}$ denotes the centralizer of h.

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Let us now consider the quiver projection of the $(4,4)$ SCFT$_2$. Since the Hilbert space of $(4,4)$ theory (3.3) (or (3.4)) is defined as the Fock space of free fields with various boundary conditions (3.2), it is enough to define the suitable projections [14] for each of the free fields with these conditions. To this aim it may be convenient to rewrite the indices (AI) $(A=0,\ldots,Q_1-1,\ I=0,\ldots,Q_5-1)$ as $((a,j)(i,j'))$, where $A=q_1a+j$ $(a=0,\ldots,q_2-1, j=0,\ldots,q_1-1)$, $I=q_1i+j'$ $(i=0,\ldots,q_3-\ 1, j'=0,\ldots,q_1-1)$. Define the complex coordinates $Z^1=x^2+ix^3$, $Z^2=x^4+ix^5$ and set $\mathcal{Z}=\left(\begin{smallmatrix} Z^1 & -\bar{Z}^2 \\ Z^2 & \bar{Z}^1 \end{smallmatrix}\right)$. The actions of the isometry group $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ along (x^2,x^3,x^4,x^5)-directions ($\sim S^3$) are realized by

$$(g_L,g_R)\in \mathrm{SU}(2)_L\times \mathrm{SU}(2)_R : \mathcal{Z}\ \longmapsto g_L\mathcal{Z}g_R\,. \tag{3.6}$$

In particular, let us consider the $\mathbb{Z}_{q_1}\subset \mathrm{SU}(2)_L$ action generated by $g_L=\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{smallmatrix}\right)$ $(\omega\equiv e^{2\pi i/q_1})$, in other words

$$Z^1\longmapsto \omega Z^1,\qquad Z^2\longmapsto \omega^{-1}Z^2\,. \tag{3.7}$$

As was discussed in several papers (see for example [34]), this $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ is no other than the R-symmetry group of $(4,4)$ SCFT$_2$ considered above. Therefore, we can expect that "orbifolding"[3] by this $\mathbb{Z}_{q_1}$-action breaks the SUSY of left-mover completely.

Under these preparations the quiver projection associated to the orbifold group $\mathbb{Z}_{q_1}$ should be defined as

$$X^{a\dot{a}}_{((a,j+n),(i,j'+n))}=X^{a\dot{a}}_{((a,j),(i,j'))}\,, \tag{3.8}$$
$$\psi^{\alpha\dot{a}}_{((a,j+n),(i,j'+n))}=\omega^{n\alpha}\psi^{\alpha\dot{a}}_{((a,j),(i,j'))}\,, \tag{3.9}$$
$$\bar{\psi}^{\dot{\alpha}\dot{a}}_{((a,j+n),(i,j'+n))}=\bar{\psi}^{\dot{\alpha}\dot{a}}_{((a,j),(i,j'))}\,. \tag{3.10}$$

It is not so difficult to check that this projection actually breaks SUSY down from $(4,4)$ to $(0,4)$. We have no supercharges of left-mover after this projection is performed. Obviously the bosonic and fermionic degrees of freedom decrease to $1/q_1$ by them, and hence we have the central charge $c=6(Q_1Q_5/q_1)\equiv 6q_1q_2q_3$ as expected. This evaluation is also consistent with the Brown-Hennaux's formula $c=3l/2G_3$, since $(1/G_3)$ $(\propto \mathrm{Vol}(S^3))$ becomes $1/q_1$ by the $\mathbb{Z}_{q_1}$-orbifoldization of S^3.

Of course, all of the twisted boundary conditions (3.2) are not necessarily compatible with the conditions of quiver projection (3.8), (3.9), and (3.10). To clarify this point it may be useful to recast further the CP indices as follows

$$\big((a,j),(i,j')\big)\equiv(\tilde{j},a,i;n)\equiv(\mathbf{a};n)\,, \tag{3.11}$$

where we set $\tilde{j}\equiv j-j'\in\mathbb{Z}_{q_1}$ and $n\equiv j'$. We also introduced the abbreviated

[3]The term "orbifolding" here is somewhat inaccurate, as was remarked in [20]. We will later discuss this point carefully, especially relating it with the analysis of multi-particle states.

notation $\mathbf{a} \equiv (\tilde{j}, a, i) \in \mathbb{Z}_{q_1} \times \mathbb{Z}_{q_2} \times \mathbb{Z}_{q_3}$. In terms of this notation we can rewrite the conditions of projections (3.8), (3.9), and (3.10) as

$$X^{a\dot{a}}_{(\mathbf{a};n+m)} = X^{a\dot{a}}_{(\mathbf{a};n)}\,, \tag{3.12}$$

$$\psi^{\alpha\dot{a}}_{(\mathbf{a};n+m)} = \omega^{m\alpha}\psi^{\alpha\dot{a}}_{(\mathbf{a};n)}\,, \tag{3.13}$$

$$\bar{\psi}^{\dot{\alpha}\dot{a}}_{(\mathbf{a};n+m)} = \bar{\psi}^{\dot{\alpha}\dot{a}}_{(\mathbf{a};n)}\,. \tag{3.14}$$

Let $\mathcal{S}$ be the subgroup of $S_{Q_1Q_5}$ composed of all the elements of the form $(g, \mathbf{m})$ ($g \in S_{q_1q_2q_3}$, $\mathbf{m} \equiv \{m_{\mathbf{a}}\} \in (\mathbb{Z}_{q_1})^{q_1q_2q_3}$), which naturally act on the CP indices in the next way

$$(g, \mathbf{m}) : (\mathbf{a}; n) \longmapsto \big(g(\mathbf{a}); n + m_{g(\mathbf{a})}\big)\,. \tag{3.15}$$

Namely, we have $\mathcal{S} \equiv S_{q_1q_2q_3} \ltimes (\mathbb{Z}_{q_1})^{q_1q_2q_3}$ (the symbol "$\ltimes$" standing for the semi-direct product indicates the relation $aba^{-1}b^{-1} \in (\mathbb{Z}_{q_1})^{q_1q_2q_3}$ for $\forall a \in S_{q_1q_2q_3}$, $\forall b \in (\mathbb{Z}_{q_1})^{q_1q_2q_3}$). Notice that this $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-action is "local" in the sense that it acts independently for each of the $q_1q_2q_3$ numbers of CP indices $\mathbf{a} \equiv (\tilde{j}, a, i)$. This fact will become important in our later discussions about multi-particle states.

Under these preparations one can easily find that only the twisted sectors associated to $g \in \mathcal{S}$ are compatible with the quiver projection. (More precisely speaking, $\mathcal{S}$ is the maximal subgroup of $S_{Q_1Q_5}$ such that it has this property.) By the same reason $\mathcal{S}$ is the discrete symmetry group which we should "gauge" in the quiver projected theory. Therefore we should define the Hilbert space for the quiver SCFT_2 as follows

$$\mathcal{H}^{(0,4)} = \left(\bigoplus_{g\in\mathcal{S}} \mathcal{H}^{\mathrm{quiver}}_g\right)^{\mathcal{S}}\,, \tag{3.16}$$

where $\mathcal{H}^{\mathrm{quiver}}_g$ ($g \in \mathcal{S}$) is defined as the Fock space of the g-twisted free fields with the quiver projections (3.12), (3.13), and (3.14) imposed. Because the constraints (3.12) and (3.14) imply the $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-part of the symmetry group $\mathcal{S}$ trivially acts on the right-mover, the Hilbert space of right-mover is the same as that of $\mathrm{Sym}^{q_1q_2q_3}(M^4)$ SCFT_2. This fact seems consistent with the observations in previous several works [15, 24]. However, the condition for the left-moving fermion (3.13) means the $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-action on it is nontrivial. It plays an quite important role in our later analysis of spectrum.

Also in the case of $M^4 = K3$ we can likewise construct the Hilbert space of the quiver SCFT_2 at least as long as we regard $K3 \cong T^4/\mathbb{Z}_2$, and thus, we do not repeat the detailed arguments.

4. Chiral primaries of quiver $(0,4)$ $\mathbf{SCFT_2}$

In this section we consider the spectrum of chiral primaries in our quiver $(0,4)$ SCFT_2. In general the chiral primaries in the $(0,4)$ theory is obtained from the 1/4 BPS states in the $(4,4)$ theory (only the right-moving SUSY are preserved).

However, the analyses in the present paper are performed only at the symmetric orbifold point in the moduli space, which is essentially a free CFT, and all the 1/4 BPS states at this point are not always stable under marginal deformations.

To clarify this point, remember that the analysis for the case $M^4 = K3$ given in [7] counts only the 1/4 BPS states *contributing to the elliptic genus*, which is an index invariant under marginal deformations of CFT. More recently the similar analysis for $M^4 = T^4$ based on the "new SUSY index" was given in [8]. Their results imply that all the "stable" 1/4 BPS states contributing to these indices with sufficiently small R-charges (below the bound $\sim (Q_1 Q_5/4)$) should be either some descendants of $(4,4)$ chiral primaries (1/2 BPS) of the single particle type or the multi-particle states, (later we clarify the precise meanings of the "single particle" and the "multi-particle" types) and this aspect is consistent with the KK spectrum of SUGRA on $AdS_3 \times S^3$.

In fact, there are many 1/4 BPS states which do not contribute to the elliptic genus (in the case $M^4 = T^4/\mathbb{Z}_2 \cong K3$) in our orbifold Hilbert space (3.3). For example,

$$
\begin{aligned}
&\sum_{(\mathbf{a};n)} \psi^{++}_{(\mathbf{a};n)-1/2} \psi^{-+}_{(\mathbf{a};n)-1/2} |0\rangle \otimes |\bar{0}\rangle \,, \\
&\sum_{(\mathbf{a};n)} \left\{ \psi^{++}_{(\mathbf{a};n)-1/2} \psi^{--}_{(\mathbf{a};n)-1/2} - \psi^{-+}_{(\mathbf{a};n)-1/2} \psi^{+-}_{(\mathbf{a};n)-1/2} \right\} |0\rangle \otimes |\bar{0}\rangle \,, \\
&\sum_{(\mathbf{a};n)} \psi^{+-}_{(\mathbf{a};n)-1/2} \psi^{--}_{(\mathbf{a};n)-1/2} |0\rangle \otimes |\bar{0}\rangle \,,
\end{aligned}
\tag{4.1}
$$

are indeed 1/4 BPS states (and define chiral primaries in the $(0,4)$-theory, since they survive after the quiver projection), but they do not appear in the expansion of elliptic genus.[4] Since we should count only the chiral primaries in $(0,4)$ theory which are stable under deformations of moduli, it is reasonable to consider the quiver projection *only on the stable BPS states in the sense of the* $(4,4)$ *theory*, that is, the BPS states contributing to the indices mentioned above.

There may be a subtlety in this argument, since we do not directly consider some index which is invariant under marginal deformations *as* $(0,4)$ $SCFT_2$. To obtain a rigid understanding of spectrum, we will have to carry out a detailed analysis based on such an index, and to this aim we may perhaps need to study thoroughly the modular property of our $(0,4)$ model. It is beyond the scope of this paper, and we would like to discuss this problem elsewhere.

[4] In fact, in [6], the structures of the 1/4 BPS states which have $h = 1$, $j = 0$, $\bar{h} \equiv \bar{j} =$ arbitrary, and can contribute to the elliptic genus are completely examined. The BPS states considered here (4.1) do not belong to this list.

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In any case we shall assume the validity of the above arguments and go on the analysis of the chiral primaries. Under this assumption we only have to look for the candidates of the stable $(0,4)$ chiral primaries of the single particle type *among the* $\mathrm{SU}(1,1|2)_L$*-descendants of* $(4,4)$ *chiral primaries*, instead of searching for them in the total $(0,4)$ Hilbert space (3.16). This claim is based on the observations with respect to the stable 1/4 BPS states in the $(4,4)$ theory given in [7, 8] we mentioned above. We believe the spectrum given in the present paper is valid at least on the generic points on the moduli space, and it is enough for the purpose of checking the correspondence between the KK spectrum of the $5d$ SUGRA in bulk and the chiral primaries of the boundary theory.

4.1 Spectrum of single particles: untwisted sector

Let us first consider the untwisted sector in the case $M^4 = T^4$. As was already explained, the $(4,4)$ Hilbert space in the untwisted sector is defined by imposing $S_{Q_1Q_5}$-invariance on the Fock space of the free bosons and fermions $\partial X^{a\dot{a}}_{(AI)}$, $\bar{\partial} X^{a\dot{a}}_{(AI)}$, $\psi^{\alpha\dot{a}}_{(AI)}$, $\bar{\psi}^{\dot{\alpha}\dot{a}}_{(AI)}$.

The chiral primary states are defined as follows

$$\begin{aligned} L_n|\alpha\rangle &= \bar{L}_n|\alpha\rangle = 0 \quad (\forall n \geq 1)\,, \\ G^{+a}_r|\alpha\rangle &= \bar{G}^{+a}_r|\alpha\rangle = 0 \quad \left(\forall r \geq -\frac{1}{2}\right)\,, \\ G^{-a}_r|\alpha\rangle &= \bar{G}^{-a}_r|\alpha\rangle = 0 \quad \left(\forall r \geq \frac{1}{2}\right)\,, \end{aligned} \tag{4.2}$$

which corresponds to the short representations of $\mathrm{SU}(1,1|2)_L \times \mathrm{SU}(1,1|2)_R$. By this definition we inevitably have

$$L_0|\alpha\rangle = J^3_0|\alpha\rangle\,, \qquad \bar{L}_0|\alpha\rangle = \bar{J}^3_0|\alpha\rangle\,. \tag{4.3}$$

So, we can characterize the chiral primary states by the two quantum numbers $(j,\bar{j})$-spins of the two SU(2)-currents $J^{\alpha\beta}$, $\bar{J}^{\dot{\alpha}\dot{\beta}}$.

As is well known, the chiral primary fields, which corresponds to the chiral primary states by the standard operator-state correspondence in unitary CFT_2, composes a ring by means of the simple operator products that has no singularity. This ring is called "chiral ring", and from the viewpoints of AdS_3/CFT_2-duality the generators of chiral ring are identified with the single particle states [30] (see also [6]). So, let us call them the "chiral primaries of single particle type", and call the other operators in chiral ring the "multi-particle type".

By taking the $S_{Q_1Q_5}$-invariance into account, we can explicitly construct all of the chiral primaries of single particle type in the untwisted sector

$$
\begin{aligned}
|\omega^{(0,0)},(0)\rangle &= |0\rangle\otimes|\bar{0}\rangle \qquad\qquad \text{(NS-NS vacuum)}\,,\\
|\omega^{(1,0)\dot{a}},(0)\rangle &= \sum_{A,I}\psi^{+\dot{a}}_{(AI)\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(0,1)\dot{a}},(0)\rangle &= \sum_{A,I}\bar{\psi}^{+\dot{a}}_{(AI)\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(2,0)},(0)\rangle &= \sum_{A,I}\psi^{++}_{(AI)\,-1/2}\psi^{+-}_{(AI)\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(0,2)},(0)\rangle &= \sum_{A,I}\bar{\psi}^{++}_{AI\,-1/2}\bar{\psi}^{+-}_{AI\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(1,1)\dot{a}\dot{b}},(0)\rangle &= \sum_{A,I}\psi^{+\dot{a}}_{(AI)\,-1/2}\bar{\psi}^{+\dot{b}}_{(AI)\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(2,1)\dot{a}},(0)\rangle &= \sum_{A,I}\psi^{++}_{(AI)\,-1/2}\psi^{+-}_{(AI)\,-1/2}\bar{\psi}^{+\dot{a}}_{(AI)\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(1,2)\dot{a}},(0)\rangle &= \sum_{A,I}\psi^{+\dot{a}}_{(AI)\,-1/2}\bar{\psi}^{++}_{(AI)\,-1/2}\bar{\psi}^{+-}_{(AI)\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(2,2)},(0)\rangle &= \sum_{A,I}\psi^{++}_{(AI)\,-1/2}\psi^{+-}_{(AI)\,-1/2}\bar{\psi}^{++}_{(AI)\,-1/2}\bar{\psi}^{+-}_{(AI)\,-1/2}|0\rangle\otimes|\bar{0}\rangle\,.
\end{aligned}
\tag{4.4}
$$

Needless to say, these correspond to the cohomology ring of T^4.

At this stage, by perfoming the quiver projection (3.8), (3.9), and (3.10) (or (3.12), (3.13), and (3.14)) and imposing the $\mathcal{S}$-invariance, we can immediately obtain the $(0,4)$ chiral primaries in the untwisted sector. The most non-trivial part of this procedure is the requirement of the $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-invariance. This invariance has no meaning for the right-mover, since it is trivially satisfied because of (3.8) and (3.10). But, taking account of the condition for $\psi^{\alpha\dot{a}}$ (3.9), we find that only the states which are $\mathrm{SU}(2)_L$-neutral (modulo $\mathbb{Z}_{q_1}$) survives when the $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-invariance are imposed. In this way we can obtain the following chiral primaries of the $(0,4)$ theory

$$
\begin{aligned}
|\omega^{(0,0)},(0)\rangle^{(0,4)} &= |0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(0,1)\dot{a}},(0)\rangle^{(0,4)} &= \sum_{(\mathbf{a};n)}\bar{\psi}^{+\dot{a}}_{(\mathbf{a};n)-1/2}|0\rangle\otimes|\bar{0}\rangle \equiv q_1\sum_{\mathbf{a}}\bar{\psi}^{+\dot{a}}_{(\mathbf{a};0)-1/2}|0\rangle\otimes|\bar{0}\rangle\,,\\
|\omega^{(2,0)},(0)\rangle^{(0,4)} &= \sum_{(\mathbf{a};n)}\left\{\psi^{++}_{(\mathbf{a};n)-1/2}\psi^{--}_{(\mathbf{a};n)-1/2}+\psi^{-+}_{(\mathbf{a};n)-1/2}\psi^{+-}_{(\mathbf{a};n)-1/2}\right\}|0\rangle\otimes|\bar{0}\rangle\equiv\\
&\equiv q_1\sum_{\mathbf{a}}\left\{\psi^{++}_{(\mathbf{a};0)-1/2}\psi^{--}_{(\mathbf{a};0)-1/2}+\psi^{-+}_{(\mathbf{a};0)-1/2}\psi^{+-}_{(\mathbf{a};0)-1/2}\right\}|0\rangle\otimes|\bar{0}\rangle\,,
\end{aligned}
$$

$$
\begin{aligned}
|\omega^{(0,2)},(0)\rangle^{(0,4)} &= \sum_{(\mathbf{a};n)} \bar\psi^{++}_{(\mathbf{a};n)-1/2}\bar\psi^{+-}_{(\mathbf{a};n)-1/2}|0\rangle\otimes|\bar 0\rangle \equiv q_1\sum_{\mathbf{a}}\bar\psi^{++}_{(\mathbf{a};0)-1/2}\bar\psi^{+-}_{(\mathbf{a};0)-1/2}|0\rangle\otimes|\bar 0\rangle,\\
|\omega^{(2,1)\dot a},(0)\rangle^{(0,4)} &= \sum_{(\mathbf{a};n)}\Big\{\psi^{++}_{(\mathbf{a};n)-1/2}\psi^{--}_{(\mathbf{a};n)-1/2}+\psi^{-+}_{(\mathbf{a};n)-1/2}\psi^{+-}_{(\mathbf{a};n)-1/2}\Big\}\,\bar\psi^{+\dot a}_{(\mathbf{a};n)-1/2}|0\rangle\otimes|\bar 0\rangle\equiv\\
&\equiv q_1\sum_{\mathbf{a}}\Big\{\psi^{++}_{(\mathbf{a};0)-1/2}\psi^{--}_{(\mathbf{a};0)-1/2}+\psi^{-+}_{(\mathbf{a};0)-1/2}\psi^{+-}_{(\mathbf{a};0)-1/2}\Big\}\,\bar\psi^{+\dot a}_{(\mathbf{a};0)-1/2}|0\rangle\otimes|\bar 0\rangle\,,\\
|\omega^{(2,2)},(0)\rangle^{(0,4)} &= \sum_{(\mathbf{a};n)}\Big\{\psi^{++}_{(\mathbf{a};n)-1/2}\psi^{--}_{(\mathbf{a};n)-1/2}+\psi^{-+}_{(\mathbf{a};n)-1/2}\psi^{+-}_{(\mathbf{a};n)-1/2}\Big\}\times\\
&\qquad\qquad\times\bar\psi^{++}_{(\mathbf{a};n)-1/2}\bar\psi^{+-}_{(\mathbf{a};n)1/2}|0\rangle\otimes|\bar 0\rangle\equiv\\
&\equiv q_1\sum_{\mathbf{a}}\Big\{\psi^{++}_{(\mathbf{a};0)-1/2}\psi^{--}_{(\mathbf{a};0)-1/2}+\psi^{-+}_{(\mathbf{a};0)-1/2}\psi^{+-}_{(\mathbf{a};0)-1/2}\Big\}\times\\
&\qquad\qquad\times\bar\psi^{++}_{(\mathbf{a};0)-1/2}\bar\psi^{+-}_{(\mathbf{a};0)-1/2}|0\rangle\otimes|\bar 0\rangle\,. \qquad (4.5)
\end{aligned}
$$

The analysis for the case of $M^4 = K3$ is also easy. Since we now deal with only the BPS states which are stable under marginal deformations, we may simply regard as $K3 = T^4/\mathbb{Z}_2$. The constraint of $\mathbb{Z}_2$-invariance kills the odd cohomologies. Hence, in (4.5) only the states $|\omega^{(0,0)},(0)\rangle^{(0,4)}$, $|\omega^{(2,0)},(0)\rangle^{(0,4)}$, $|\omega^{(0,2)},(0)\rangle^{(0,4)}$, $|\omega^{(2,2)},(0)\rangle^{(0,4)}$ survive as the single particle chiral primaries in the untwisted sector.

It should be remarked that in the orbifold $T^4/\mathbb{Z}_2$, we further have the $\mathbb{Z}_2$-twisted sectors which correspond to the blow-up modes of 16 fixed points. In fact we must take account of them to recover the full cohomology of $K3$. However, because all the contributions from these twisted sectors are $(1,1)$-cohomologies, we get no chiral primaries from these sectors (the $\mathrm{SU}(2)_L$-modules for them do not include neutral states). Of course, this is not the case when we deal with the twisted sectors of symmetric orbifold at the same time. We will later observe how the chiral primaries corresponding to these extra $(1,1)$-forms appear in the twisted sectors of symmetric orbifold.

4.2 Spectrum of single particles: twisted sectors

The analysis for the twisted sectors is more complicated than that for the untwisted sector. As we observed before, we can suppose that each twisted sector of the symmetric orbifold corresponds to a Young tableau $(n_1,\ldots,n_l)$, and this is decomposed into a set of $\mathbb{Z}_{n_i}$-twisted sectors, each of which can describe a single-particle state with a large value of R-charge (see for example [30]).

Since we are here concerned about single particle states, we only have to learn how to describe the $\mathbb{Z}_p$-twisted sector which corresponds to the blow-up modes of one of the orbifold singularities such that $X^{a\dot a}_{(A_0I_0)} = \cdots = X^{a\dot a}_{(A_{p-1}I_{p-1})}$. It is known that this can be elegantly but somewhat abstractedly described on some Fock space [35]. Namely, one can prepare the bosonic and fermionic oscillators (in Ramond sector) associated to all the even and odd cohomology elements of M^4 respectively.

The oscillators of the type $\alpha_{-1}(\omega)$ ($\forall\omega \in H^*(M^4)$) correspond to the untwisted sector and those of the type $\alpha_{-p}(\omega)$ describe the $\mathbb{Z}_p$-twisted sector. Although this description has many excellences, it may be more useful for our purpose to describe "concretely" the chiral primaries in terms of the σ-model variables themselves.

We must introduce the "$\mathbb{Z}_p$-twisted string" or the "long string with length p" to describe this sector.[5] We label the objects in this sector by the index $\mathcal{A} \equiv [(A_0I_0),\ldots,(A_{p-1}I_{p-1})]$, $A_i = 0,\ldots,Q_1-1$, $I_i = 0,\ldots,Q_5-1$, such that $(A_iI_i) \neq (A_jI_j)$ for $\forall i \neq j$. The long string coordinates for this sector (defined on the world-sheet $0 \leq \sigma \leq 2\pi$ which is rescaled from the p-times one $0 \leq \sigma \leq 2\pi p$) is given as follows ($\Phi$ means each field $X^{a\dot{a}}$, $\psi^{\alpha\dot{a}}$, $\bar{\psi}^{\dot{\alpha}\dot{a}}$)

$$\Phi_{\mathcal{A}}(\sigma) = \Phi_{(A_rI_r)}(p\sigma - 2\pi r)\,, \qquad \left(\frac{2\pi r}{p} \leq \sigma \leq \frac{2\pi(r+1)}{p}\right). \tag{4.6}$$

These variables $X^{a\dot{a}}_{\mathcal{A}}$, $\psi^{\alpha\dot{a}}_{\mathcal{A}}$, $\bar{\psi}^{\dot{\alpha}\dot{a}}_{\mathcal{A}}$ can produce the superconformal currents of the central charge $c = 6$ in the same way as the untwisted sector (3.1). However, the condition of $\mathbb{Z}_p$-invariance which should be imposed on the physical Hilbert space leaves us only the modes $n \in p\mathbb{Z}$ for the bosons and the Ramond fermions, and the modes $n \in p\left((1/2)+\mathbb{Z}\right)$ for the NS fermions. By this moding out, we can obtain the superconformal currents $\{\hat{L}_n, \hat{G}^{\alpha a}_r, \hat{J}^{\alpha\beta}\}$ properly describing this $\mathbb{Z}_p$-twisted sector [38, 39] (Here we only write the NS sector.)

$$\begin{aligned}
\hat{L}_n &= \frac{1}{p}L_{np} + \frac{p^2-1}{4p}\delta_{n0}\,,\\
\hat{G}^{\alpha a}_r &= \begin{cases} \dfrac{1}{\sqrt{p}}G^{\alpha a(\mathrm{NS})}_{pr} & (p = 2k+1) \\ \dfrac{1}{\sqrt{p}}G^{\alpha a(\mathrm{R})}_{pr} & (p = 2k), \end{cases}\\
\hat{J}_n &= J_{np}\,.
\end{aligned} \tag{4.7}$$

These generate the $N = 4$ SCA with $c = 6p$. The anomaly term in the expression of $\hat{L}_n$ essentially corresponds to the Schwarzian derivative of the conformal mapping $z \longmapsto z^p$).

Observing these definitions (4.7), we can find that the NS vacuum $|0,\mathcal{A}\rangle$ of $\mathbb{Z}_p$-twisted sector (corresponding to the twist field which creates a cut on the world-sheet by the state-operator correspondence) should possess the following properties

- $p = 2k+1$

$$\begin{aligned}
\hat{L}_0|0,\mathcal{A}\rangle &= \frac{p^2-1}{4p}|0,\mathcal{A}\rangle\,,\\
\hat{J}^3_0|0,\mathcal{A}\rangle &= 0\,.
\end{aligned} \tag{4.8}$$

[5] In this paper we are using the term "long string" in the sense which is different from that given in [19, 37].

- $p = 2k$

$$\hat{L}_0|0,\mathcal{A}\rangle = \left(\frac{p^2-1}{4p} + \frac{1}{4p}\right)|0,\mathcal{A}\rangle \equiv \frac{p}{4}|0,\mathcal{A}\rangle\,,$$
$$\hat{J}_0^3|0,\mathcal{A}\rangle = -\frac{1}{2}|0,\mathcal{A}\rangle\,. \tag{4.9}$$

As one can find in (4.7), when p is even, the NS supercurrent $\hat{G}_r^{\alpha a}$ is made up of the Ramond one before imposing the $\mathbb{Z}_p$-invariance. The extra vacuum energy "$1/4p$" and the extra R-charge "$-1/2$" in the case of $p = 2k$ originate from this fact.

The chiral primaries for this $\mathbb{Z}_p$-twisted sector are defined by the conditions

$$\begin{aligned}
\hat{L}_n|\alpha\rangle &= \bar{\hat{L}}_n|\alpha\rangle = 0 \quad (\forall n \geq 1)\,,\\
\hat{G}_r^{+a}|\alpha\rangle &= \bar{\hat{G}}_r^{+a}|\alpha\rangle = 0 \quad \left(\forall r \geq -\frac{1}{2}\right)\,,\\
\hat{G}_r^{-a}|\alpha\rangle &= \bar{\hat{G}}_r^{-a}|\alpha\rangle = 0\,, \quad \left(\forall r \geq \frac{1}{2}\right)\,.
\end{aligned} \tag{4.10}$$

It may be slightly surprizing that the NS vacuum $|0,\mathcal{A}\rangle$ itself is *not* a chiral primary according to this conditions (4.10). By recalling the definitions of supercurrents $\hat{G}_r^{\alpha a}$ (4.7), we can find out that we must "fill" the NS vacuum with all the fermionic oscillators $\psi_{\mathcal{A}r}^{+\dot{a}}$ with $r < p/2$ to make it satisfy the conditions (4.10). Namely, all the possible chiral primaries in the $\mathbb{Z}_p$-twisted sector are written in the following form;

$$|\omega^{(q,\bar{q})},(p-1)\rangle = \sum_{\mathcal{A}} |\omega^q,\mathcal{A}\rangle \otimes \overline{|\omega^{\bar{q}},\mathcal{A}\rangle}\,, \tag{4.11}$$

where the summation should be taken over all the possible $\mathbb{Z}_p$-twisted strings labeled by $\mathcal{A}$, and the left(right)-moving parts are defined by

- $p = 2k+1$

$$\begin{aligned}
|\omega^0,\mathcal{A}\rangle &= \prod_{i=0}^{k-1} \psi^{++}_{\mathcal{A}-((1/2)+i)}\psi^{+-}_{\mathcal{A}-((1/2)+i)}|0,\mathcal{A}\rangle\,,\\
|\omega^{1\dot{a}},\mathcal{A}\rangle &= \psi^{+\dot{a}}_{\mathcal{A}-(p/2)}|\omega^0,\mathcal{A}\rangle\,,\\
|\omega^2,\mathcal{A}\rangle &= \psi^{++}_{\mathcal{A}-(p/2)}\psi^{+-}_{\mathcal{A}-(p/2)}|\omega^0,\mathcal{A}\rangle\,,
\end{aligned} \tag{4.12}$$

- $p = 2k$

$$\begin{aligned}
|\omega^0,\mathcal{A}\rangle &= \prod_{i=0}^{k-1} \psi^{++}_{\mathcal{A}-i}\psi^{+-}_{\mathcal{A}-i}|0,\mathcal{A}\rangle\,,\\
|\omega^{1\dot{a}},\mathcal{A}\rangle &= \psi^{+\dot{a}}_{\mathcal{A}-(p/2)}|\omega^0,\mathcal{A}\rangle\,,\\
|\omega^2,\mathcal{A}\rangle &= \psi^{++}_{\mathcal{A}-(p/2)}\psi^{+-}_{\mathcal{A}-(p/2)}|\omega^0,\mathcal{A}\rangle\,.
\end{aligned} \tag{4.13}$$

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It is easy to check that[6]

$$\hat{L}_0|\omega^{(q,\bar{q})},(p-1)\rangle = \hat{J}_0^3|\omega^{(q,\bar{q})},(p-1)\rangle = \frac{q+p-1}{2}|\omega^{(q,\bar{q})},(p-1)\rangle\,,$$
$$\hat{\bar{L}}_0|\omega^{(q,\bar{q})},(p-1)\rangle = \hat{\bar{J}}_0^3|\omega^{(q,\bar{q})},(p-1)\rangle = \frac{\bar{q}+p-1}{2}|\omega^{(q,\bar{q})},(p-1)\rangle\,. \tag{4.14}$$

Now, let us consider the reduction to the $(0,4)$ theory. To observe how the quiver projection works on the twisted sectors, it may be useful to recast the long string variables $\Phi_{\mathcal{A}}$ defined by (4.6) as follows (Again $\Phi_{\mathcal{A}}$ means $X^{a\dot{a}}_{\mathcal{A}}$, $\psi^{\alpha\dot{a}}_{\mathcal{A}}$, $\bar{\psi}^{\dot{\alpha}\dot{a}}_{\mathcal{A}}$)

$$\Phi^{(s)}_{\mathcal{A}}(\sigma) \stackrel{\mathrm{def}}{=} \sum_{m\in\mathbf{Z}} \Phi_{\mathcal{A}pm+s} e^{i(m+(s/p))\sigma} \quad (s=0,\ldots,p-1)\,, \tag{4.15}$$

where we assume the mode expansions of $\Phi_{\mathcal{A}}$ is

$$\Phi_{\mathcal{A}}(\sigma) = \sum_{m\in\mathbf{Z}} \Phi_{\mathcal{A}m} e^{im\sigma}\,. \tag{4.16}$$

(As we already observed, when $p=2k+1$, the fermionic fields $\psi^{\alpha\dot{a}}_{\mathcal{A}}$, $\bar{\psi}^{\dot{\alpha}\dot{a}}_{\mathcal{A}}$ should have the half-integral mode expansions. It is straightforward to generalize our discussions here to this case, and we omit it.) It is obvious that the set of "twisted fields" $\{\Phi^{(s)}_{\mathcal{A}}\}_{s=0,\ldots,p-1}$ describes the same degrees of freedom as $\Phi_{\mathcal{A}}$ does. These should have the monodromies

$$\Phi^{(s)}_{\mathcal{A}}(\sigma+2\pi) = \omega_p^s \Phi^{(s)}_{\mathcal{A}}(\sigma) \quad \left(\omega_p \equiv e^{2\pi i/p}\right)\,, \tag{4.17}$$

and this implies the following formula of the Fourier transformation

$$\Phi^{(s)}_{\mathcal{A}}(\sigma) = \sum_{r=0}^{p-1} \Phi_{(A_r I_r)}(\sigma)\omega_p^{-rs}\,,$$
$$\Phi_{(A_r I_r)}(\sigma) = \frac{1}{p}\sum_{s=0}^{p-1} \Phi^{(s)}_{\mathcal{A}}(\sigma)\omega_p^{rs}\,. \tag{4.18}$$

As we showed in the previous section, in the quiver projected theory only the $g\in\mathcal{S}$ twisted sectors are possible. Furthermore, according to our assumption, we only have to consider the projection on the $\mathrm{SU}(1,1|2)_L$-descendants of $(4,4)$ chiral primaries. These lead to the fact that we only have to deal with the twisted sectors of the type $\mathcal{A} \equiv [(A_0I_0),\ldots,(A_{p-1}I_{p-1})] \equiv [(\mathbf{a}_0;n_0),\ldots,(\mathbf{a}_{p-1};n_{p-1})]$, where $\mathbf{a}_i \in \mathbb{Z}_{q_1}\times\mathbb{Z}_{q_2}\times\mathbb{Z}_{q_3}$, $n_i\in\mathbb{Z}_{q_1}$ and, among other things, $\mathbf{a}_i\neq\mathbf{a}_j$ $(\forall i\neq j)$ holds.

Let us next discuss the $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-invariance. Let $\mathbf{m}\equiv\{m_{\mathbf{a}}\}\in(\mathbb{Z}_{q_1})^{q_1q_2q_3}$, and we define the action of $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$ on the CP indices of twisted sectors as follws

$$\sigma_{\mathbf{m}}(\mathcal{A})(\equiv(1,\mathbf{m})(\mathcal{A})) = \left[(\mathbf{a}_0;n_0+m_{\mathbf{a}_0}),\ldots,(\mathbf{a}_{p-1};n_{p-1}+m_{\mathbf{a}_{p-1}})\right]\,, \tag{4.19}$$

[6] This means that the chiral primary state $|\omega^{(q,\bar{q})},(p-1)\rangle$ is identified with the element "$\alpha_{-p}(\omega^{(q,\bar{q})})|0\rangle$" of the fock space translating the cohomologies of the Hilbert scheme [35, 30].

where $\mathcal{A} \equiv [(\mathbf{a}_0; n_0), \ldots, (\mathbf{a}_{p-1}; n_{p-1})]$ as above. This acts trivially on $X_{\mathcal{A}}^{\alpha\dot{a}}$ and $\bar{\psi}_{\mathcal{A}}^{\dot{\alpha}\dot{a}}$, but acts on $\psi_{\mathcal{A}}^{\alpha\dot{a}}$ in a non-trivial manner. To see it explicitly, it is easier to consider the twisted fields $\{\psi_{\mathcal{A}}^{\alpha\dot{a}(s)}\}_{s=0,\ldots,p-1}$ than $\psi_{\mathcal{A}}^{\alpha\dot{a}}$ itself, and we can obtain from the condition (3.9)

$$\psi_{\sigma_{\mathbf{m}}(\mathcal{A})}^{\alpha\dot{a}(s)} = \frac{1}{p} \sum_{s'=0}^{p-1} \sum_{r=0}^{p-1} \psi_{\mathcal{A}}^{\alpha\dot{a}(s')} \omega_p^{r(s'-s)} \omega_{q_1}^{\alpha m_{\mathbf{a}_r}} . \tag{4.20}$$

In the simplest example such that $m_{\mathbf{a}_0} = \cdots = m_{\mathbf{a}_{p-1}} = m$, this reduces to

$$\psi_{\sigma_{\mathbf{m}}(\mathcal{A})}^{\alpha\dot{a}} = \omega^{m\alpha} \psi_{\mathcal{A}}^{\alpha\dot{a}} , \tag{4.21}$$

and we also obtain for the NS vacuum $|0, \mathcal{A}\rangle$

$$\begin{aligned} |0, \sigma_{\mathbf{m}}(\mathcal{A})\rangle &= |0, \mathcal{A}\rangle \quad (p = 2k+1) , \\ |0, \sigma_{\mathbf{m}}(\mathcal{A})\rangle &= \omega^{-m} |0, \mathcal{A}\rangle \quad (p = 2k) . \end{aligned} \tag{4.22}$$

Recall that, when p is odd, $|0, \mathcal{A}\rangle$ is $\mathrm{SU}(2)_L$-neutral, but it has an extra $\mathrm{SU}(2)_L$-charge $-1/2$ for the cases with even p. The conditions (4.21) and (4.22) again imply that only the states possessing the $\omega^{2J_0^3}$-invariance survive after performing the quiver projection.

In summary the chiral primaries of the twisted sectors in $(0,4)$ theory can be constructed by the following procedures:

- First, we should consider all the (single particle) chiral primaries in the twisted sectors of the forms $\mathcal{A} = [(\mathbf{a}_0; n_0), \ldots, (\mathbf{a}_{p-1}; n_{p-1})]$, $\mathbf{a}_i \neq \mathbf{a}_j$ $(\forall i \neq j)$, which are essentially the twisted sectors in $\mathrm{Sym}^{q_1 q_2 q_3}(M^4)$-$\mathrm{SCFT}_2$ (*not* in $\mathrm{Sym}^{Q_1 Q_5}(M^4)$).
- Second, consider $\mathrm{SU}(1,1|2)_L$-module over each of them, and pick up the primary states such that

$$\omega^{2J_0^3} |\alpha\rangle = |\alpha\rangle , \tag{4.23}$$

  which are the desired chiral primary states in our $(0,4)$-theory.

Since we already know the explicit forms of chiral primaries of $(4,4)$-theory, it is straightforward to write down those for $(0,4)$-theory according to the above prescription. However, to avoid unessential complexity, we here only present the spectrum of degeneracy of them with certain quantum numbers. They correspond to the short representations of $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SU}(1,1|2)_R$, and we express each of them by $(\bar{j}; s)$, where

$$\begin{aligned} \bar{J}_0^3 |\alpha\rangle (\equiv \bar{\hat{L}}_0 |\alpha\rangle) &= \bar{j} |\alpha\rangle \quad (R\text{-charge}) , \\ (\hat{L}_0 - \bar{\hat{L}}_0) |\alpha\rangle &= s |\alpha\rangle \quad (\text{spin}) . \end{aligned} \tag{4.24}$$

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in the same way as in SUGRA. Moreover, we express the space of $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SU}(1,1|2)_R$ irreps. in which lowest component is a boson (fermion) by $\mathcal{H}^{(+)}_{\mathrm{SCFT}}$ ($\mathcal{H}^{(-)}_{\mathrm{SCFT}}$) as before.

Here we give a comment before we exhibit the spectrum of chiral primaries. By the above prescription we should take account of not only the states with $J^3_0 = 0$ but also with $J^3_0 = q_1 k/2$ ($\forall k \in \mathbb{Z}_{\geq 0}$). However, the states possessing non-zero J^3_0-charges correspond to very massive modes in string theory. In fact, in the original $AdS_3 \times S^2$-geometry this charge is identified with the winding along x^5-direction (in our T-dualized framework, the KK momentum along the Taub-NUT circle), and hence the states with this winding are much heavier than the KK excitations on S^3 (We are now assuming these winding modes have the masses of the extent of Planck mass, and the KK excitations on S^3 have the masses $\sim 1/l^2 \ll M^2_{\mathrm{Planck}}$.), which means these states are negligible in the region where SUGRA is valid.

Notice also that our quiver SCFT_2 describes the branes at the fixed point of the orbifold $\mathbb{C}^2/\mathbb{Z}_{q_1}$, *which is the decompactification limit of the Taub-NUT space*. So, the stringy heavy states just mentioned become lighter under this orbifold limit. The existence of these extra states with non-zero J^3_0-charges in our quiver SCFT_2 does not mean a failure of the AdS_3/CFT_2-correspondence.

Therefore, we here only present the spectra of the "light" chiral primaries with vanishing J^3_0-charges, which should be compared with the results of SUGRA. We will give the complete spectra including non-zero J^3_0-charges in Appendix, since they have rather involved forms.

By including the contributions from all of the untwisted and twisted sectors, we can obtain the following spectrum for $M^4 = T^4$ (We set $D \equiv q_1 q_2 q_3$ again)

$$\begin{aligned}
\mathcal{H}^{(+)}_{\mathrm{SCFT}}(T^4) &= (0;2) + 5(0;1) + (0;0) + (1;2) + 15(1;1) + 14(1;0) + (1;-1) + \\
&\quad + \sum_{n=2}^{[(D-1)/2]} \left\{(n;2) + 15(n;1) + 15(n;0) + (n;-1)\right\} + \\
&\quad + \begin{cases} (N+1;1) + 6(N+1;0) + (N+1;-1) & (D = 2N+1) \\ 9(N;1) + 14(N;0) + (N;-1) & (D = 2N), \end{cases} \\
\mathcal{H}^{(-)}_{\mathrm{SCFT}}(T^4) &= 6\left(\frac{1}{2};\frac{3}{2}\right) + 14\left(\frac{1}{2};\frac{1}{2}\right) + \\
&\quad + \sum_{n=2}^{[(D-1)/2]} \left\{6\left(n-\frac{1}{2};\frac{3}{2}\right) + 20\left(n-\frac{1}{2};\frac{1}{2}\right) + 6\left(n-\frac{1}{2};-\frac{1}{2}\right)\right\} + \\
&\quad + \begin{cases} 2\left(N+\frac{1}{2};\frac{3}{2}\right) + 16\left(N+\frac{1}{2};\frac{1}{2}\right) + 6\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D = 2N+1) \\ 4\left(N+\frac{1}{2};\frac{1}{2}\right) + 4\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D = 2N). \end{cases}
\end{aligned} \tag{4.25}$$

It is worth mentioning that the R-charge in this spectrum has upper bound $\sim D/2$. This is because the length of maximally twisted string becomes $D \equiv q_1 q_2 q_3$ (*not* $Q_1 Q_5$) after performing the quiver projection.

Let us now turn to the studies for $M^4 = K3$. As in the previous discussions in the untwisted sector, we regard as $K3 \cong T^4/\mathbb{Z}_2$.

For the untwisted sector in the sense of $\mathbb{Z}_2$-orbifold, we need to make little change in the above results for $M^4 = T^4$. The requirement of $\mathbb{Z}_2$-invariance simply kills the odd cohomologies of T^4, and we obtain the contributions from the $(4,4)$ chiral primaries of the following types $|\omega^{(0,0)}, (p-1)\rangle$, $|\omega^{(2,0)}, (p-1)\rangle$, $|\omega^{(0,2)}, (p-1)\rangle$, $|\omega^{(2,2)}, (p-1)\rangle$ and also the four series of the type $|\omega^{(1,1)}, (p-1)\rangle$.

To get the complete spectrum we must also make the twisted sectors of $\mathbb{Z}_2$-orbifold join in our game. There are 16 twisted sectors corresponding to each fixed points of $T^4/\mathbb{Z}_2$. We here only focus on one of them, because all of them have the same algebraic structure. At the same time we take the $\mathbb{Z}_p$-twisted sector in the sense of the symmetric orbifold.

- $p = 2k+1$

  We have the next mode expansions for the bosonic and fermionic oscillators; $\alpha^{a\dot{a}}_{\mathcal{A},(n+1/2)}$, $\psi^{\alpha\dot{a}}_{\mathcal{A},n}$, $(n \in \mathbb{Z})$, and for the NS vacuum $|0,\mathcal{A}\rangle$ (We omit the label which distinguishes the 16 sectors for simplicity.) we have

$$\begin{aligned} \hat{L}_0|0,\mathcal{A}\rangle &= \left(\frac{p^2-1}{4p} + \frac{1}{2p}\right)|0,\mathcal{A}\rangle \equiv \frac{p^2+1}{4p}|0,\mathcal{A}\rangle\,, \\ \hat{J}^3_0|0,\mathcal{A}\rangle &= -\frac{1}{2}|0,\mathcal{A}\rangle\,. \end{aligned} \tag{4.26}$$

- $p = 2k$

  We have the mode expansions $\alpha^{a\dot{a}}_{\mathcal{A},(n+1/2)}$, $\psi^{\alpha\dot{a}}_{\mathcal{A},(n+1/2)}$, $(n \in \mathbb{Z})$, and for the NS vacuum

$$\begin{aligned} \hat{L}_0|0,\mathcal{A}\rangle &= \left(\frac{p^2-1}{4p} + \frac{1}{4p}\right)|0,\mathcal{A}\rangle \equiv \frac{p}{4}|0,\mathcal{A}\rangle\,, \\ \hat{J}^3_0|0,\mathcal{A}\rangle &= 0\,. \end{aligned} \tag{4.27}$$

In these expressions the extra zero-point energies and R-charges assigned to the NS vacuua have their origins in these twisted mode expansions. The superconformal currents in this twisted sector (that is, the "twisted" in the meaning of both $T^4/\mathbb{Z}_2$ and the symmetric orbifold) are also defined as before (4.7), and they give us only one chiral primary state for each sector, which has the form $|\omega^{(1,1)}, (p-1)\rangle$

$$|\omega^{(1,1)}, (p-1)\rangle = \sum_{\mathcal{A}} |\omega^1, \mathcal{A}\rangle \otimes \overline{|\omega^1, \mathcal{A}\rangle}\,, \tag{4.28}$$

$$|\omega^1, \mathcal{A}\rangle = \prod_{i=0}^{k-1} \psi^{++}_{\mathcal{A}-i} \psi^{+-}_{\mathcal{A}-i} |0, \mathcal{A}\rangle\,, \quad (p = 2k+1)$$

$$|\omega^1, \mathcal{A}\rangle = \prod_{i=0}^{k} \psi^{++}_{\mathcal{A}-((1/2)+i)} \psi^{+-}_{\mathcal{A}-((1/2)+i)} |0, \mathcal{A}\rangle\,, \quad (p = 2k)\,. \tag{4.29}$$

We can easily check that they have the expected quantum numbers

$$\hat{L}_0 |\omega^1, \mathcal{A}\rangle = \hat{J}_0^3 |\omega^1, \mathcal{A}\rangle = \frac{p}{2} |\omega^1, \mathcal{A}\rangle\,. \tag{4.30}$$

Of course, there are 16 independent such states corresponding to each twisted sectors of $T^4/\mathbb{Z}_2$, and they compensate the missing cohomologies of $\mathrm{Sym}^{Q_1 Q_5}(K3)$. Thus we can get the 20 sequences of the form $|\omega^{(1,1)}, (p-1)\rangle$ by including also the 4 contributions from the untwisted sector.

The next step is the same as that in the case of T^4. We can take the quiver projection for this twisted sector and can obtain the spectrum which is modded out by $\omega^{2J_0^3}$. Gathering the contributions from all the sectors, we can lastly obtain the next spectrum for $K3$

$$\begin{aligned}
\mathcal{H}^{(+)}_{\mathrm{SCFT}}(K3) &= (0;2) + (0;1) + (0;0) + (1;2) + 23(1;1) + 22(1;0) + (1;-1) + \\
&+ \sum_{n=2}^{[(D-1)/2]} \big\{(n;2) + 23(n;1) + 23(n;0) + (n;-1)\big\} + \\
&+ \begin{cases} (N+1;1) + 2(N+1;0) + (N+1;-1) & (D = 2N+1) \\ 21(N;1) + 22(N;0) + (N;-1) & (D = 2N)\,, \end{cases} \\
\mathcal{H}^{(-)}_{\mathrm{SCFT}}(K3) &= 2\left(\frac{1}{2};\frac{3}{2}\right) + 42\left(\frac{1}{2};\frac{1}{2}\right) + \\
&+ \sum_{n=2}^{[(D-1)/2]} \left\{2\left(n-\frac{1}{2};\frac{3}{2}\right) + 44\left(n-\frac{1}{2};\frac{1}{2}\right) + 2\left(n-\frac{1}{2};-\frac{1}{2}\right)\right\} + \\
&+ \begin{cases} 42\left(N+\frac{1}{2};\frac{1}{2}\right) + 2\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D = 2N+1) \\ 2\left(N+\frac{1}{2};\frac{1}{2}\right) + 4\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D = 2N)\,. \end{cases}
\end{aligned} \tag{4.31}$$

To end this subsection we present a comparison with the results of SUGRA given in the previous section. By comparing the results (2.7), (2.8), (4.25), and (4.31), we can notice that both are equivalent except the following slight differences.

First, we now have the upper bound for R-charges, which is essentially the one first remarked in [30] ("stringy exclusion principle"). This difference is not problematic at all, as far as we work in the sufficiently large $D \equiv q_1 q_2 q_3$ assumed in the Maldacena conjecture.

The second difference is owing to the existence of the chiral primaries $|\omega^{(0,2)}, (0)\rangle$, $|\omega^{(2,0)}, (0)\rangle$ in the $(4,4)$ theory. They correspond to the degrees of freedom which essentially exists only on the boundary (called "singleton") [24, 6]. Thus we have no contradiction even if we cannot find them in the bulk theory.

In this way we can conclude that the spectrum of chiral primaries of our quiver SCFT_2 is consistent with that of the SUGRA in the $AdS_3 \times S^2$-geometry at least for the single particle states. We must further study the multi-particle states to claim the complete agreement between them. The next subsection will devote to this subject.

4.3 Discussions about multi-particle states

In the previous subsection we have shown that our quiver SCFT_2 can produce the spectrum of single particle states which is consistent with that of SUGRA on $AdS_3 \times S^2$. The essential part of our discussion was that the quiver projection works on the Hilbert space as moding out by $\omega^{2J_0^3}$. At this point one might ask the following questions.

Is the quiver projection truly needed? Why can't we define the Hilbert space of the boundary $(0,4)\mathrm{SCFT}_2$ *by directly imposing the constraints* $\omega^{2J_0^3}|\alpha\rangle = |\alpha\rangle$ *on the Hilbert space of* $(4,4)\mathrm{SCFT}_2$ *on* $\mathrm{Sym}^{Q_1Q_5}(M^4)$*?*

The answer to the first question is *yes*, and we can answer to the second question as follows: one of the evidences for us to need the quiver projection is given by evaluating the degrees of freedom. If we naively mode out by $\omega^{2J_0^3}$ to define the $(0,4)$ theory (To get a more suitable candidate of the $(0,4)$ theory, one should further incorporate the twisted sectors of this orbifoldization to ensure the modular invariance), we will obtain the $(0,4)\mathrm{SCFT}_2$ with the same central charge $c = 6Q_1Q_5$ as that of $(4,4)$ theory. But we want the SCFT_2 with the $1/q_1$ times degrees of freedom. As was already seen, the quiver projection actually yields the $(0,4)$ SCFT_2 with the central charge $c = 6Q_1Q_5/q_1 \equiv 6q_1q_2q_3$.

The second reason why we need to consider the quiver projection is more crucial and has a deep relationship to the analysis of the multi-particle states. Roughly speaking, in order to be consistent with the spectrum of SUGRA, the multi-particle states in the boundary SCFT_2 should have the structure $\sim \mathcal{O}_1\mathcal{O}_2\cdots|0\rangle$, where $\mathcal{O}_i$ denotes some descendant of the chiral primary of single particle type. As was already mentioned, in [7, 8] it is shown that, in the $(4,4)\mathrm{SCFT}_2$ on $\mathrm{Sym}^{Q_1Q_5}(M^4)$, general BPS states which contribute to the invariant index (elliptic genus for $K3$, the index given in [8] for T^4) should have indeed such forms, if their R-charges are less than $((Q_1Q_5+1)/4) \sim (Q_1Q_5/4)$. In other words the correspondence of the chiral primaries between the $AdS_3 \times S^3$-SUGRA and the boundary $(4,4)\mathrm{SCFT}_2$ are satisfactory in the level of multi-particle states as far as taking the large Q_1Q_5-limit.

Now, we first try to define the Hilbert space of the $(0,4)$ theory, which we express by $\mathcal{H}^{(0,4)'}$ in order to distinguish it from $\mathcal{H}^{(0,4)}$ (3.16) defined by the quiver projection, by simply imposing the constraints $\omega^{2J_0^3}|\alpha\rangle = |\alpha\rangle$ on the Hilbert space of $(4,4)$ theory $\mathcal{H}^{(4,4)}$ (Here, we overlook the problem about the central charge we already mentioned.) As we observed before, the $\mathcal{H}^{(0,4)'}$ so defined includes all the single particle states with correct quantum numbers. But, how about the multi-particles? If we admit the complete agreement between the spectrum of $(4,4)\mathrm{SCFT}_2$ and the $AdS_3 \times S^3$-SUGRA, and adopt $\mathcal{H}^{(0,4)'}$ as the Hilbert space of $(0,4)$-theory, we will obtain the following general forms of the BPS states of $(0,4)$ SCFT_2

$$|\alpha\rangle = \mathcal{O}_1\mathcal{O}_2\cdots|0\rangle\,, \qquad [2J_0^3, \mathcal{O}_i\} = r_i\mathcal{O}_i\,, \quad \sum_i r_i \in q_1\mathbb{Z}\,, \tag{4.32}$$

where $\mathcal{O}_i$ denotes the 1/4 BPS observables of the single particle type in the sense of $(4,4)$ SUSY, and the reader should read $[\,,\}$ as the (anti-)commutator for the bosonic (fermionic) field $\mathcal{O}_i$. The important point is that each r_i need not be a multiple of q_1, although $|\alpha\rangle$ must be $\mathbb{Z}_{q_1}$-invariant. Such $|\alpha\rangle$ is of course a multi-particle states in the sense of the $(4,4)$ theory, but generally *not* in the $(0,4)$ theory, because the constituents $\mathcal{O}_i$ may not belong to the Hilbert space $\mathcal{H}^{(0,4)'}$.

On the other hand, the Hilbert space of multi-particle states in SUGRA should be defined by taking the (symmetric or anti-symmetric) tensor products of single particle states. This fact brings us the result that the SUGRA on $AdS_3 \times S^3/\mathbb{Z}_{q_1}$ should include only the multi-particle states of the following type

$$|\alpha\rangle = \mathcal{O}_1\mathcal{O}_2\cdots|0\rangle\,, \qquad [2J_0^3, \mathcal{O}_i\} = r_i\mathcal{O}_i\,, \quad (\forall r_i \in q_1\mathbb{Z})\,. \tag{4.33}$$

Clearly the states with the form (4.33) merely spans the subspace of that for (4.32). We thus face the existence of many missing states, if we adopt the simple prescription of orbifoldization by $\omega^{2J_0^3}$!

Next let us observe how we can resolve this inconsistency by considering the quiver projection. We should notice that each of single-particle operators has the next CP index structure; $\mathcal{O}_i = \sum_{\mathcal{A}^{(i)}} \mathcal{O}_{i,\mathcal{A}^{(i)}}$, where $\mathcal{A}^{(i)} \equiv [(\mathbf{a}_0^{(i)}; n_0^{(i)}), \ldots, (\mathbf{a}_{p_i-1}^{(i)}; n_{p_i-1}^{(i)})]$ labels one of the $\mathbb{Z}_{p_i}$-twisted sector. (If $p_i = 1$, one should understand it as the untwisted sector, of course.) The general multi-particles in the $(4,4)$ theory have the forms;

$$|\alpha\rangle = \mathcal{O}_1\mathcal{O}_2\cdots|0\rangle \equiv \left(\sum_{\mathcal{A}^{(1)}} \mathcal{O}_{1,\mathcal{A}^{(1)}}\right)\left(\sum_{\mathcal{A}^{(2)}} \mathcal{O}_{2,\mathcal{A}^{(2)}}\right)\cdots|0\rangle\,. \tag{4.34}$$

The quiver projection again brings us the requirements of the $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-invariance with respect to the CP indices. Especially, we can take an element $\mathbf{m} \equiv \{m_{\mathbf{a}}\}$ of $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$ such that $m_0^{(i)} = \cdots = m_{p_i-1}^{(i)} = m^{(i)}$, where $m_r^{(i)} \equiv m_{\mathbf{a}_r^{(i)}}$. It is the most important to notice that *we can choose these $m^{(i)} \in \mathbb{Z}_{q_1}$ independently for each i, because of the "locality" of the $(\mathbb{Z}_{q_1})^{q_1q_2q_3}$-symmetry.* Therefore, according to the discussions given in the previous subsections, we find that the $\omega^{2J_0^3}$-invariance are *separately* imposed on each of the single particle components $\mathcal{O}_i$, in contrast to the usual orbifoldization of CFT. In other words, $\mathcal{H}^{(0,4)}$ only includes the BPS states possessing the forms (4.33), although $\mathcal{H}^{(0,4)'}$ also includes those of the forms (4.32).

In this way we can conclude that the quiver projection does reproduce the consistent spectrum of multi-particles (4.33). We can also rephrase: *the orbifoldization of bulk SUGRA does not correspond to the orbifoldization of the boundary CFT, rather correspond to the quiver projection of it!*

5. Summary and comments

In this paper we have proposed a candidate for the suitable boundary SCFT_2 which is dual to the SUGRA on $AdS_3 \times S^2 \times M^6$ ($M^6 = T^6$, $T^2 \times K3$) by the quiver techniques. We presented a detailed analysis on the chiral primaries, including all of the twisted sectors of symmetric orbifolds, and obtained the spectrum which is consistent with SUGRA in the Maldacena limit.

It is plausible and may not be so surprising that the quiver projection on the single particle states leads to the simple moding out prescription by $\omega^{2J_0^3}$. But for the multi-particle states, this procedure of orbifolding leads to the wrong results. We must carefully consider the quiver projection, not the orbifolding, to get a consistent spectrum of the multi-particles.

However, because of just this fact that our quiver SCFT_2 is not an usual orbifold SCFT_2, it will become a subtle problem whether it has actually the modular invariance. Needless to say, the modular invariance is an important consistency condition of CFT_2 on torus, and in addition, in the thermodynamical studies on the AdS_3-gravity including some BTZ black-holes [40], it is expected to yield a very powerful tool [4, 30]. The check of modular invariance for the quiver SCFT_2 seems a challenging and an important problem, and we hope to investigate this problem in future.

One of the natural generalizaitons of the studies in this paper is the works for $M^6 = CY_3$. Although the analyses for the completely general CY_3 seem very difficult, it might be possible to apply our formulation at least to some elliptic CY_3. We can reduce by the "fiber-wise T-duality" the original system of M5 to the one composed of a D3 wrapped around a SUSY 2-cycle in the base manifold of the elliptic fibration, located at the singularity of an orbifold.

It may be also interesting to compare our quiver SCFT_2 with the "space-time CFT" [12] given in the string theoretical approach along the line of [18]. It is worth remarking that the quiver projection can completely reproduce the $(0,4)$ version of the "stringy exclusion principle" of [30] (That is, the unitarity bound of the arbitrary chiral primary is $j \leq q_1q_2q_3 \, (\equiv Q_1Q_5/q_1)$, and that for the single particle is $j \leq q_1q_2q_3/2$. The latter was already mentioned, and the former is easily shown by counting the degrees of freedom and by taking account of the fermi statistics.). Whether this bound can be also explained in the framework of [12] is an important test. To answer this question completely, we may need first to study carefully the unitarity bound for the $(4,4)$ space-time CFT [18], maybe based on the discussions given in [41]. To this aim the approach from the Matrix string theory [42] may become more efficient than that of the "old fashioned" string on AdS_3 back-ground. Some studies along this line will be presented in [37].

Acknowledgments

I would like to thank T. Eguchi for valuable discussions, and especially should appreciate T. Kawano and K. Okuyama for their collaboration in the early stage of this work.

This work is partly supported by the Grant-in-Aid for Scientific Research on Priority Area 707 "Supersymmetry and Unified Theory of Elementary Particles", Japan Ministry of Education.

A. Complete spectra of the chiral primaries in quiver $\mathbf{SCFT_2}$

Here we give the complete spectra of the chiral primaries including the general states such that $2J_0^3|\alpha\rangle = kq_1|\alpha\rangle$ $(k \in \mathbb{Z}_{\geq 0})$. To this aim it is convenient to introduce the symbol $(\bar{j}; s)_k$ for the irreducible $\mathrm{SL}(2,\mathbb{R})_L \times$ $\mathrm{SU}(1,1|2)_R$-modules in which all the states should satisfy

$$2J_0^3|\alpha\rangle = kq_1|\alpha\rangle\,, \tag{A.1}$$

and the highest weight state (chiral primary) does

$$\bar{L}_0|\alpha\rangle = \bar{J}_0^3|\alpha\rangle = \left(\bar{j} + \frac{kq_1}{2}\right)|\alpha\rangle\,, \qquad (L_0 - \bar{L}_0)|\alpha\rangle = s|\alpha\rangle\,. \tag{A.2}$$

Under this preparation we can write down the desired spectra of chiral primaries as follows ($D \equiv q_1q_2q_3$ as before)

- $M^4 = T^4$

$$\begin{aligned}
\mathcal{H}^{(+)}_{\mathrm{SCFT}}(T^4) = &\sum_{k=0}^{q_2q_3-1}\big\{(0;2)_k + 5(0;1)_k + (0;0)_k + (1;2)_k + 6(1;1)_k + (1;0)_k\big\} + \\
&+\sum_{k=0}^{q_2q_3}\big\{9(1;1)_k + 13(1;0)_k + (1;-1)_k\big\} + \\
&+\sum_{n=2}^{[(D-1)/2]}\ \sum_{k=0}^{[(D-2n)/q_1]}\big\{(n;2)_k + 15(n;1)_k + 15(n;0)_k + (n;-1)_k\big\} + \\
&+\begin{cases}(N+1;1) + 6(N+1;0) + (N+1;-1) & (D = 2N+1)\\ 9(N;1) + 14(N;0) + (N;-1) & (D = 2N)\,,\end{cases}
\end{aligned} \tag{A.3}$$

$$\mathcal{H}^{(-)}_{\rm SCFT}(T^4) = \sum_{k=0}^{q_2q_3-1}\left\{4\left(\frac{1}{2};\frac{3}{2}\right)_k + 4\left(\frac{1}{2};\frac{1}{2}\right)_k\right\} + \sum_{k=0}^{q_2q_3}\left\{2\left(\frac{1}{2};\frac{3}{2}\right)_k + 10\left(\frac{1}{2};\frac{1}{2}\right)_k\right\} +$$
$$+ \sum_{n=2}^{\left[\frac{(D-1)}{2}\right]}\sum_{k=0}^{\left[\frac{(D-2n)}{q_1}\right]}\left\{6\left(n-\frac{1}{2};\frac{3}{2}\right)_k + 20\left(n-\frac{1}{2};\frac{1}{2}\right)_k + 6\left(n-\frac{1}{2};-\frac{1}{2}\right)_k\right\} +$$
$$+ \begin{cases} 2\left(N+\frac{1}{2};\frac{3}{2}\right) + 16\left(N+\frac{1}{2};\frac{1}{2}\right) + 6\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D=2N+1) \\ 4\left(N+\frac{1}{2};\frac{1}{2}\right) + 4\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D=2N)\,. \end{cases} \tag{A.4}$$

- $M^4 = K3$

$$\mathcal{H}^{(+)}_{\rm SCFT}(K3) = \sum_{k=0}^{q_2q_3-1}\left\{(0;2)_k + (0;1)_k x + (0;0)_k + (1;2)_k + 2(1;1)_k + (1;0)_k\right\} +$$
$$+ \sum_{k=0}^{q_2q_3}\left\{21(1;1)_k + 21(1;0)_k + (1;-1)_k\right\} +$$
$$+ \sum_{n=2}^{[(D-1)/2]}\sum_{k=0}^{[(D-2n)/q_1]}\left\{(n;2)_k + 23(n;1)_k + 23(n;0)_k + (n;-1)_k\right\} +$$
$$+ \begin{cases} (N+1;1) + 2(N+1;0) + (N+1;-1) & (D=2N+1) \\ 21(N;1) + 22(N;0) + (N;-1) & (D=2N)\,. \end{cases} \tag{A.5}$$

$$\mathcal{H}^{(-)}_{\rm SCFT}(K3) = \sum_{k=0}^{q_2q_3-1}\left\{2\left(\frac{1}{2};\frac{3}{2}\right)_k + 2\left(\frac{1}{2};\frac{1}{2}\right)_k\right\} + \sum_{k=0}^{q_2q_3}40\left(\frac{1}{2};\frac{1}{2}\right)_k +$$
$$+ \sum_{n=2}^{\left[\frac{(D-1)}{2}\right]}\sum_{k=0}^{\left[\frac{(D-2n)}{q_1}\right]}\left\{2\left(n-\frac{1}{2};\frac{3}{2}\right)_k + 44\left(n-\frac{1}{2};\frac{1}{2}\right)_k + 2\left(n-\frac{1}{2};-\frac{1}{2}\right)_k\right\} +$$
$$+ \begin{cases} 42\left(N+\frac{1}{2};\frac{1}{2}\right) + 2\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D=2N+1) \\ 2\left(N+\frac{1}{2};\frac{1}{2}\right) + 2\left(N+\frac{1}{2};-\frac{1}{2}\right) & (D=2N)\,. \end{cases} \tag{A.6}$$

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